

The shear-induced alpha-effect and long-term variations in solar dynamo

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ABSTRACT

The consequences of the shear-induced alpha effect to the long-term modulation of magnetic activity are examined with the help of the axisymmetric numerical dynamo model that includes the self-consistent description of the angular momentum balance, heat transport and magnetic field generation in the spherical shell. We find that the shear contributions to alpha effect can complicate the long-term behaviour of the large-scale magnetic activity and differential rotation in nonlinear dynamo. Additionally we consider the impact secular magnetic activity variations to the secular modulations of the solar luminosity and radius.

1 INTRODUCTION

It seems that the long-term variations of the sunspot's activity and phenomena similar to the Maunder minimum are widely presented on the late-type stars like the Sun, see e.g.(25; 1). Generally, it may be useful to distinguish the different kinds of time variations of magnetic activity on the stars for two types. One is irregular disappearance of sunspot cycle similarly to what was happened during the Maunder minimum at the second half of the 17-th century, (4). Another kind of variations can be attributed to the quasi-regular grand activity cycles. The grand activity cycles are natural to explain as a result of the nonlinear interactions between magnetic fields and the main sources of the solar dynamo, which are the differential rotation and convection, (6; 29; 27; 19; 8). The given nonlinear mechanisms may be responsible for the chaotic variations of magnetic activity on the late type stars and on the Sun, as well. Though, the chaotic behaviour was demonstrated only in the case of the highly supercritical dynamos, (6; 27; 8). Currently, some doubts are cast upon the possibility for the supercritical regimes in the stellar dynamo. The main constraint for feasibility of the supercritical dynamo is due to magnetic helicity conservation. However, the problem can be overcome if to allow the helicity outflux of the dynamo region. Another constraint comes into the play if to consider the distributed dynamo. In this case the energy of the generated large-scale magnetic fields (LSMF) is hardly to exceed the equipartition level with the energy of convective flow. These arguments motivate us to look for mechanisms which could induce the long-term variations for such a weakly non-linear dynamo regimes.

In this paper we consider an effect that can provide a nonlinear coupling between two main sources of the solar dynamo, these are the differential rotation and α effect. In according to (12; pip05; 18) the shear (differential rotation) changes both the structure and the amplitude of the alpha

effect. For the Sun the most important component of alpha effect is $\alpha_{\phi\phi}$. The estimation of shear contribution to the $\alpha_{\phi\phi}$ with regards to the typical conditions in the solar CZ shows that it gives only about 15% of the whole magnitude. In the nonlinear dynamo the LSMF are included in the angular momentum balance inside CZ. In fact the grand activity cycle may be considered as a result of relaxation of the perturbations angular momentum balance in CZ emerged due to LSMF (via the Λ -quenching(6; 19), or the large-scale Lorenz force(13; 26; 8)). In this case the incorporation of the shear-induced alpha effect can complicate the relaxation process because it supplies the additional destabilization the sources of magnetic field generation due to the time-dependence of the alpha effect on the differential rotation distribution in CZ.

Below, we demonstrate the significance of the shear-induced alpha-effect for the long-term evolution of magnetic activity using the numerical model of the axisymmetric dynamo in the convective shell. The model includes the numerical solution for the dynamo equations, angular momentum balance and heat transport problem with regards for the physical conditions that take place in the main part of the solar CZ. In the next part of the paper we give the short description of the model (more details can be found in the article(17)). The results of numerical simulations are discussed at the third part of the paper.

2 THE MODEL DESIGN

The basic physical quantities under considerations are the induction vector of the axisymmetric LSMF, $\mathbf{B} = \mathbf{e}_\phi B(r, \theta) + \text{curl} \left(\frac{\mathbf{e}_\phi A(r, \theta)}{r \sin \theta} \right)$, the large-scale flow - differential rotation, $\mathbf{V} = \mathbf{e}_\phi r \sin \theta \Omega(r, \theta)$ (we neglect the meridional circulation, here), the mean specific entropy, $S(r, \theta)$, and the pressure, $P(r, \theta)$. The magnetic activity and differen-

tial rotation give rise to perturbations of the thermodynamic stage of SCZ. The thermodynamic deviations from reference model (taken on the base of solar interior model from Stix(28)) is described with help of the mean entropy production equation. In following to Pipin & Kitchatinov(20) we write it as

$$\rho T \frac{\partial S}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r} r^2 F_r - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin^2 \theta F_\theta \quad (1)$$

$$+ \left(T_{\theta\phi} \frac{\partial \Omega}{\partial \theta} + r T_{r\phi} \frac{\partial \Omega}{\partial r} \right) \sin \theta - \frac{1}{4\pi} (\mathcal{E} \cdot \mathbf{rot} \mathbf{B}),$$

Again, for details please look at paper(17) and references there. The heat flux \mathbf{F} is a sum of convective and radiative fluxes $\mathbf{F} = \mathbf{F}_c + \mathbf{F}_{rad}$, where in convective flux we take into account both the influence of rotation and magnetic fields. The second string in (1) is responsible for the heating and cooling of the matter in CZ due to generation and dissipation of large-scale magnetic fields and differential rotation there.

The angular momentum transport in convective shell is described with,

$$\rho r \sin \theta \frac{\partial \Omega}{\partial t} = \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \left(T_{r\phi} + \frac{F_L B}{4\pi r^2 \sin \theta} \frac{\partial A}{\partial \theta} \right) \quad (2)$$

$$+ \frac{1}{r \sin^2 \theta} \frac{\partial}{\partial \theta} \sin^2 \theta \left(T_{\theta\phi} - \frac{F_L B}{4\pi r \sin \theta} \frac{\partial A}{\partial r} \right),$$

$$T_{r\phi} = \rho P_\nu \chi_T \sin \theta \left[\Phi_\perp \Psi_1(\Omega^*, \beta) r \frac{\partial \Omega}{\partial r} \right. \\ + \left(\Phi_\parallel \Psi_2(\Omega^*, \beta) - \Phi_\perp \Psi_2(\Omega^*, \beta) \right) \times \\ \times \cos \theta \left(r \cos \theta \frac{\partial \Omega}{\partial r} - \sin \theta \frac{\partial \Omega}{\partial \theta} \right) \\ \left. - \left(\frac{\alpha_M}{\gamma} \right)^2 \Omega \left(\mathcal{J}_0 - \mathcal{J}_1 + c_V \sin^2 \theta \mathcal{J}_1 \right) \phi_V(\Omega^*, \beta) \right]$$

$$T_{\theta\phi} = \rho P_\nu \chi_T \sin \theta \left[\Phi_\perp \Psi_1(\Omega^*, \beta) \sin \theta \frac{\partial \Omega}{\partial \theta} \right. \\ + \left(\Phi_\parallel \Psi_2(\Omega^*, \beta) - \Phi_\perp \Psi_1(\Omega^*, \beta) \right) \times \\ \times \sin^2 \theta \left(\sin \theta \frac{\partial \Omega}{\partial \theta} - r \cos \theta \frac{\partial \Omega}{\partial r} \right) \\ \left. - \left(\frac{\alpha_M}{\gamma} \right)^2 \Omega \cos \theta \left(\mathcal{J}_2 \phi_H(\Omega^*, \beta) + c_V \sin^2 \theta \mathcal{J}_1 \phi_V(\Omega^*, \beta) \right) \right].$$

The turbulent stresses, $T_{(r,\theta)\phi}$, describe the effective redistribution of the angular momentum in rotating convective shell due to convection. The measure of rotational influence on convection is the Coriolis number, $\Omega^* = 2\Omega_0 \tau_c$; here Ω_0 is the basic rotation rate of a star, and τ_c is the typical convective turnover time. The parameter β measures the influence of magnetic field on convection. It is defined as $\beta = |B| / (\sqrt{4\pi\rho} u_c)$, where u_c is rms convective velocity. Both τ_c and u_c are taken from the reference solar interior model by Stix(2002). Functions, $\Phi_{(\perp,\parallel)}$, $\mathcal{J}_{(1,2)}$ are responsible for the influence of rotation on the turbulent viscosity and on the Λ effect respectively. They are given in papers(5; 9). The effect of magnetic fields on viscosity and on the Λ effect (which is for brevity call as Λ -quenching) is approximated with help of $\Psi_{1,2}$ and with $\psi_{(V,H)}$ respectively. They are defined as follows. Its definitions can be found in appendix. In case of the slow rotation we have $\Psi_{(1,2)}(\Omega^*, \beta) = \psi_{(1,2)}(\beta)$, $\psi_{(V,H)}(\Omega^*, \beta) = K(\beta)_{(1,2)}$ where $\psi_{(1,2)}$ are given in ... and $K_{(1,2)}$ in ... The c_V is a parameter

to adjust the differential rotation law in shell in more close agreement with helioseismology results. We use $c_V = 1.3$ to get nearly radial profiles of angular velocity isolines in the main part of CZ (see below). $P_\nu = \frac{\nu_T}{\chi_T}$ is the effective Prandtl number and χ_T , ν_T - typical eddy heat conductivity and eddy viscosity respectively. In simulations presented here we used $P_\nu = 0.3, 0.1, 0.05$. These gives examples for two different regimes. The effects of Λ -quenching prevails the Lorentz force for $P_\nu = 0.3$ and the opposite case is for $P_\nu = 0.05$. Also we have to introduce the parameter $F_L < 1$ to tune the power of the large-scale Lorentz force. Its necessity is determined by shortcomings in dynamo model. Particularly, in our model the relation between the typical strength of toroidal and poloidal components of the large-scale magnetic field is $B_T/B_P \approx 50$ which is less than can be expected from observations, where $B_T/B_P > 100$.

We use the following dynamo equations in a spherical coordinate system,

$$\frac{\partial B}{\partial t} = \frac{1}{r} \left\{ \frac{\partial (\Omega, A)}{\partial (r, \theta)} + \frac{\partial r \mathcal{E}_\theta}{\partial r} - \frac{\partial \mathcal{E}_r}{\partial \theta} \right\}, \quad (3)$$

$$\mathcal{E}_r = \chi_T P'_m \left[B \left(-\frac{2\alpha_M \phi_1}{\ell_c \gamma} + \frac{\psi_\eta \phi_\parallel}{r} \right) \sin \theta \cos \theta \right. \\ - \frac{\psi_\eta (\phi + \phi_\parallel \sin^2 \theta)}{r \sin \theta} \frac{\partial \sin \theta B}{\partial \theta} + \phi_1 \psi_\eta \sin \theta \cos \theta \frac{\partial B}{\partial r} \\ + C_\alpha \frac{\alpha_M \phi_\alpha}{\ell_c \gamma} ((\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_4 \cos^2 \theta) \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial A}{\partial \theta} \\ \left. + (\mathcal{A}_2 + \mathcal{A}_4 \cos^2 \theta) \frac{1}{r} \frac{\partial A}{\partial r} \right],$$

$$r \mathcal{E}_\theta = \chi_T P'_m \left[B \left(r (\phi_2 - 2\phi_2 \sin^2 \theta) \frac{\alpha_M}{\ell_c \gamma} - \phi_\parallel \psi_\eta \cos^2 \theta \right) \right. \\ + \psi_\eta (\phi + \phi_\parallel \cos^2 \theta) \frac{\partial r B}{\partial r} - \psi_\eta \phi_\parallel \sin \theta \cos \theta \frac{\partial B}{\partial \theta} \\ + C_\alpha \frac{\alpha_M \phi_\alpha}{\ell_c \gamma} ((\mathcal{A}_2 + \mathcal{A}_4 \cos^2 \theta) \frac{1}{r^2} \frac{\partial A}{\partial \theta} \\ \left. + (\mathcal{A}_1 + \mathcal{A}_4 \sin^2 \theta) \frac{\cos \theta}{r \sin \theta} \frac{\partial A}{\partial r} \right],$$

$$\frac{\partial A}{\partial t} = \chi_T P'_m \psi_\eta \left[(\phi + \phi_\parallel \cos^2 \theta) \frac{\partial^2 A}{\partial r^2} \right. \quad (4) \\ + \frac{\phi + \phi_\parallel \sin^2 \theta}{r^2} \frac{\partial}{\partial \theta} \frac{1}{\sin \theta} \frac{\partial A}{\partial \theta} + \\ + \phi_\parallel \left(\frac{\sin^2 \theta}{r} \frac{\partial A}{\partial r} + \frac{3 \sin \theta \cos \theta}{r^2} \frac{\partial A}{\partial \theta} - \frac{\sin 2\theta}{r} \frac{\partial^2 A}{\partial r \partial \theta} \right) \\ - \frac{\alpha_M}{\ell_c \gamma} ((\cos^2 \theta (\phi_1 - \phi_3) + \phi_1 + \phi_2) \frac{\partial A}{\partial r} \\ - (\phi_1 - \phi_3) \frac{\sin 2\theta}{2r} \frac{\partial A}{\partial \theta}) \left. \right] + C_\alpha \chi_T P'_m \frac{\alpha_M}{\ell_c \gamma} \psi_\alpha \left[\mathcal{A}_1 \cos \theta \right. \\ + \Psi_{\alpha 1} \cos \theta \sin^2 \theta \frac{\partial \log(\Omega)}{\partial \log(r)} \left. \right]$$

$$+ \sin(\theta) \frac{\partial \log(\Omega)}{\partial \theta} (\Psi_{\alpha 2} + \cos^2(\theta) \Psi_{\alpha 1}) \Big] r \sin \theta B,$$

More details about the given dynamo equations can be found in ... Below, we make some commentaries about (3,4). The power of generation sources due to alpha effect is characterized via $C_\alpha = \frac{\eta_0}{\alpha_0 R_\odot}$, where η_0 and α_0 are the maxima of radial distributions of magnetic diffusivity and alpha effect in CZ; $\alpha_M = \frac{\ell_c}{H_p}$ is a mixing-length parameter, which relates the pressure scale height H_p and typical mixing length of convective flows ℓ_c ; P'_m relates the eddy diffusivity and eddy heat conductivity $\eta_T = P'_m \chi_T$. To match the period cycle we used $P'_m = 0.05$ in what follows. Functions $\phi_{n,\parallel}$, A_n and $\Psi_{\alpha n}$ serve to determine the dependence of the turbulent drift, dissipation and generation of magnetic fields on the Coriolis number. They are discussed in ...]. The magnetic quenching function for magnetic diffusivity and transport effects is ψ_η . The α quenching is described with ϕ_α . Both are given in appendix. The eddy viscosity is determined with help of mixing-length relations for rms convective velocity u_c , $\chi_T = \frac{u_c \ell_c}{3}$, $u_c = \frac{\ell_c}{2} \sqrt{-\frac{g}{c_p} \frac{\partial S}{\partial r}}$.

2.1 Boundary conditions

The system (2,3,4,1) is integrated on the volume domain which is confined on the radius in $r = [0.715 - 0.96] R_\odot$ and on the polar angle - from pole to pole. For the angular momentum balance problem we define the stress-free conditions both at the bottom and at the top of CZ via $T_{r\phi} + \frac{B}{4\pi r^2 \sin \theta} \frac{\partial A}{\partial \theta} = 0$. For dynamo equations, at the bottom of CZ we put a superconductivity condition. At the surface we assume the condition of the “partial reflection” of magnetic energy suggested previously by Kitchatinov et al. ..., i.e., for vector potential A the vacuum conditions are applied and the azimuthal field B is related with poloidal component of electromotive force via $\mathcal{E}_\theta = -\frac{\eta_T B^2}{R_\odot B_{(0)}}$, where we take $B_{(0)} = 200$ Gs. For the heat transfer problem, at the bottom we take the vanishing convective flux and at the surface the thermal flux is approximated as effective black-body radiation: $F_r = \frac{\mathcal{L}_\odot}{4\pi r^2} \left(1 + 4 \frac{T_e \delta S}{c_p T_{eff}}\right)$, where T_{eff} is the effective temperature of the photosphere and T_e is the temperature at the outer boundary of the integration domain. The reference stratification inside domain is assumed nearly adiabatic. It is fixed if to specify the temperature T_e and density ρ_e at the outer boundary. We use the parameters from the solar interior model by Stix, $T_e = 1.93 \times 10^5$ K and $\rho_e = 3.6 \times 10^{-3} \text{ g cm}^{-3}$.

3 RESULTS AND DISCUSSION.

Before discussion results it remains to fix some of parameters introduced above. The mixing-length parameter is specified to $\alpha_M = 1.3$. The low magnetic Prandtl number $P_m = 0.05$ helps to match the period of magnetic cycle in the model. Note, the maximum of radial heat conductivity

in our model is $\chi_T = 2.1 \times 10^{13} \text{ cm}^3 \text{ s}^{-1}$ so the correspondent magnetic diffusivity is about $10^{12} \text{ cm}^3 \text{ s}^{-1}$. The threshold C_α was obtained via numerical trials. As an initial condition we choose the weak magnetic field with zero parity index (see the definition below). The dipole parity activates first for $C_\alpha^{cr} \geq 0.66$. For the Prandtl number $P_\nu = 1$ we did not find the long cycle within range $0.66 < C_\alpha < 1$. Note, the choice $C_\alpha \geq 1$ would mean that we are going to inject more energy in alpha-effect than it is available from convective flows. The long cycles activate after increase of the hydrodynamical response time on the angular momentum balance perturbations by magnetic fields. For analyzing purpose it is useful to introduce the following global quantities of solution. The overall magnetic activity is quantified by integral of modulus of magnetic flux contained in CZ, $F_M = \int_0^\pi \int_{r_b}^{r_e} |B(r, \theta)| r \sin \theta dr d\theta$. The parity index - $P = \frac{E^{(S)} - E^{(A)}}{E^{(S)} + E^{(A)}}$, where $E^{(S)}$, $E^{(A)}$ are the integrated over convection zone energies of the quadrupole and dipole components of magnetic fields, respectively.

The main subject of the paper concerns with the influence of the shear contributions to alpha effect on the long-term variations in solar dynamo. The system (2,3,4,1) forms the solid self-consistent basis for investigation of this problem with numerical simulations. Previous studies show that interaction of the LSMF and differential rotation is, perhaps, the most important factor that is responsible for excitation of the secular magnetic activity cycles in the non-linear stellar dynamo. Coupling the LSMF generation and angular momentum transport is described by (2,3,4). The changes of the heat transport in CZ due to LSMF activity and differential rotation result in modifications both in the dynamo processes and in the angular momentum balance. The influence of these modifications to the secular magnetic activity variations needs separate study. We will not address this problem here. At the end we show some preliminary results about influence of the LSMF and differential rotation variations on the changes in solar irradiance and radius. Just below we consider results where the heat transport problem (1) is fixed at the beginning of time-stepping procedure by solving (2,1) without regards for the LSMF.

The Fig.1 shows some results of numerical simulations in the case of $C_\alpha = 0.85$. On the left side of the Fig.1 we put the results concerning the variations of the integral magnetic flux F_M (top), parity P (middle) and latitudinal shear (bottom). The shear is quantified by the difference of angular velocity between equator and poles. This difference is averaged over N and S-hemispheres. There with solid line we draw the result of simulated evolutions with regards for shear-induced alpha effect. The dashed line shows the same but with the discarded contribution of shear to alpha effect. In the picture we filtered out the short cycle to see the long-term behaviour more clearly. In both cases we see the long-term variation of the magnetic activity. They are caused (similar to (19)) primarily by the global interaction of LSMF and differential rotation. It is clearly seen that pattern of the long-term variations is much more complicated if the shear-induced alpha effect is allowed for. Though this effect is certainly not the principal factor in exciting the long-term variation. In particular, for this model the most important factors driving the long-term variations are the Λ -quenching, the large-scale Lorentz force and the non-linear

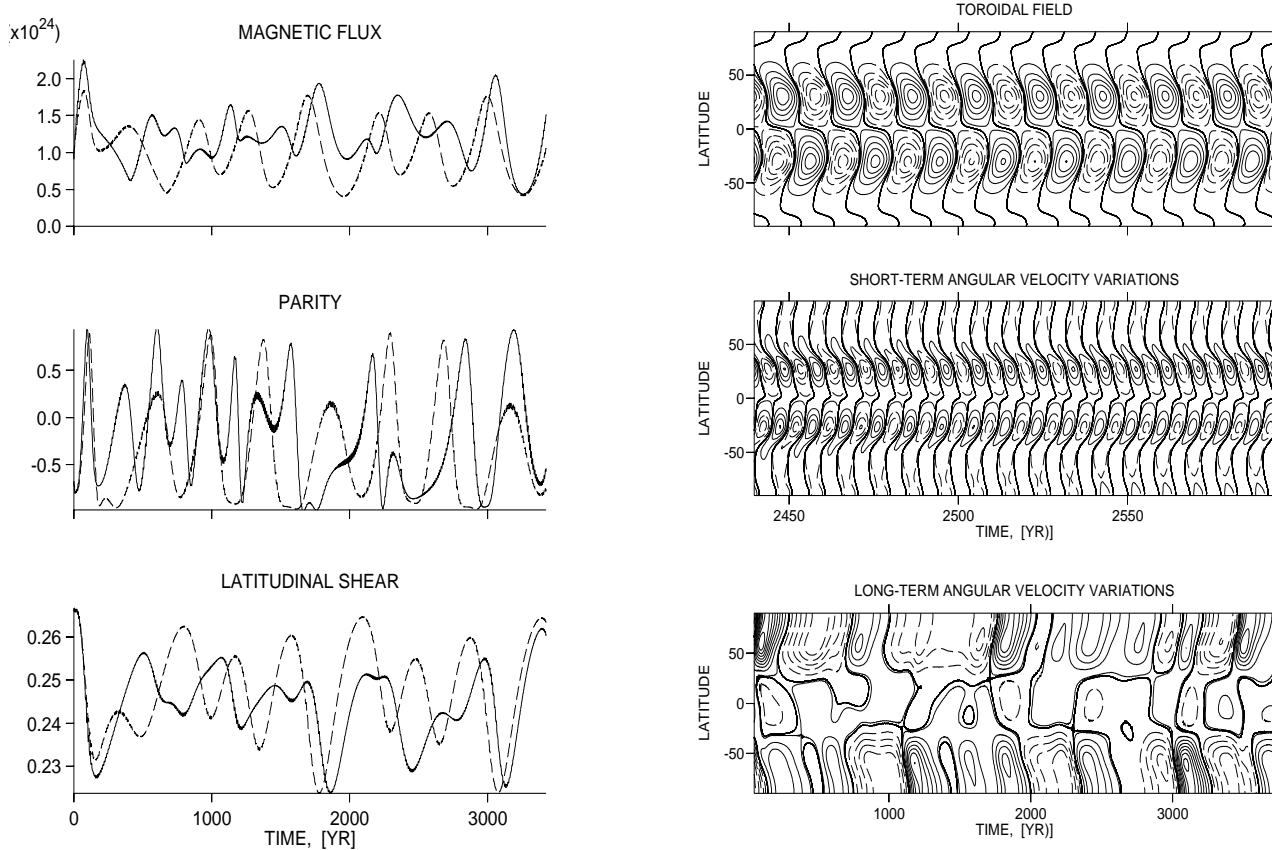


Figure 1. Variations of magnetic flux and parity (left) and the Maunder diagrams for toroidal magnetic fields and angular velocity variations (right).

magnetic diffusion. The long-term variations disappears if one of them is excluded.

On the right side of the Fig.1 we draw the butterfly diagram of toroidal LSMF near the surface for time interval about 200 years (top). The amplitude of magnetic field changes is 400 Gs. The corresponding diagram of the torsional variations of angular velocity at the surface for this period is given at the middle. The variations is measured by deviations of the current value of angular velocity from the moving mean quantity obtained with averaging on the period of cycle. The frequency of torsional variations is twice to the basic frequency of magnetic cycle. The magnitude of the deviations of rotational velocity is about 4 m/s . The diagram of the long-term variations of angular velocity spanning the whole interval of simulations is at the bottom. The short-term torsional oscillations are filtered out there. The negative deviations of velocity are shown with dashed contours and positive ones are shown with solid lines. The amplitude of secular modulation of surface angular velocity is about 40 m/s . According to the given diagramm the relatively accelerated (or slow down) zones occupy the high latitude of the solar surface and they drift to poles in course of grand activity cycle. On the picture some isolines span

from pole to pole what can be interpreted as the angular momentum exchanges between hemispheres. This confirms the global nature of the long-term variations demonstrated by our model.

At the end we discuss briefly the possible impact of the long-term variations of magnetic activity and differential rotation to the heat transport and hydrostatic balance in SCZ. The heat transport is described with eq.(1). The description of hydrostatic balance with regards for LSMF and differential rotation is given in . The contribution of magnetic field to irradiance output is estimated by the outflux of the LSMF's energy from the dynamo region. This gives the maximum amplitude of variations in solar irradiance if we assume that the energy of escaping field is completely converted to radiation. The radius variations were estimated from the changes of the zero mode of the gravitational potential. On the Fig.2 we show of the long-term variations of the integral parameters describing the integral magnetic flux and parity (top), the latitudinal shear and the relative radius variations(middle), the relative variations of luminosity and the heat outflux (bottom). The short cycles were filtered out there. The panel in the middle of Fig.2 clearly shows that variations of the latitudinal shear and solar ra-

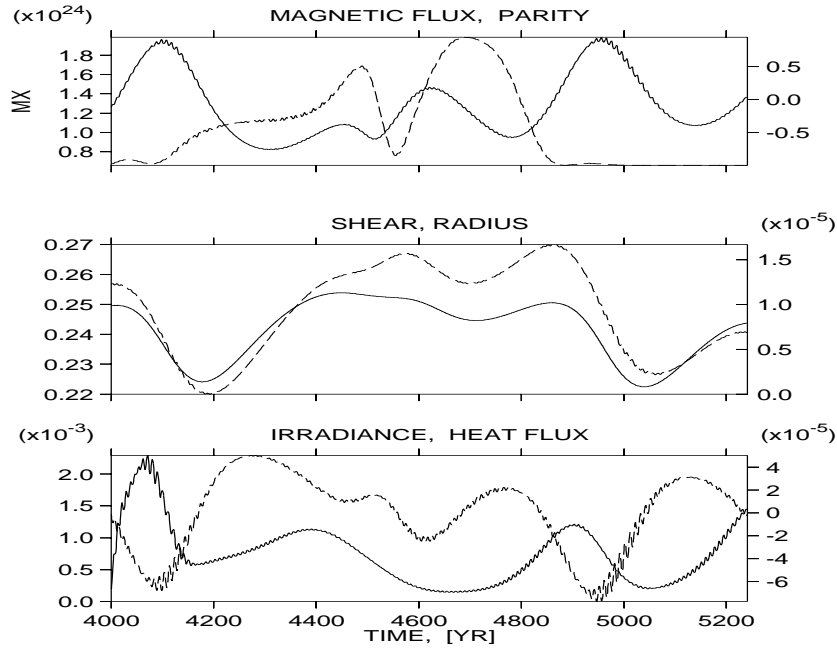


Figure 2. Long-term variations of the integral magnetic flux (top, solid line, left scale), parity (top, dashed line, right scale), latitudinal shear (middle, solid line, left scale), relative radius variations (middle, dashed line, right scale), relative variations of luminosity (bottom, solid line, left scale) and the relative heat outflux variations (bottom, dashed line, right scale).

dius go in phase, while they vary slightly ahead the changes of the magnetic activity. In our model the delay is about 100 years. It worth to note that in the short, 11-th year cycles the radius varies nearly in phase with magnetic cycle with relative amplitude about $10^{-6}R_{\odot}$. The amplitude of secular variations of radius is larger but it still rather small, $1.5 \times 10^{-5}R_{\odot}$. This model seems to confirm our previous result. In .. we concluded that the primary source of the solar radius changes could be the modulation of the centrifugal forces by magnetic activity (via modulation of differential rotation). In according to simulations the secular modulation of luminosity due to the escape of the magnetic energy from the dynamo region is about $2 \times 10^{-3}\mathcal{L}_{\odot}$. This is the *maximum possible* estimate for the LSMF input to radiation. The secular variations of the heat outflux in the model are rather small, $10^{-4}\mathcal{L}_{\odot}$. They are in anti-phase relation with magnetic activity.

In conclusion we would like to summaries the results of the paper as follows. The differential rotation modifies the standard alpha effect changing its amplitude and inducing the additional mean electromotive force that is perpendicular to original magnetic field. The dependence of the alpha effect on differential can complicate the long-term variations in non-linear dynamo. The main shortcomming of the model presented is that we ignored the helicity conservation law. Some preliminary results given in .. show that the helicity conservation could controll the largest time scale of long-term variation in the non-linear dynamo. The present model demonstrates the secular variations of radius. The secular maximum of the radius corresponds to epoch the maximal

differential rotation and it foregoes the secular maximum of magnetic activity. The amplitude of of secular variations of radius is $1.5 \times 10^{-5}R_{\odot}$. The corresponded variations of solar irradiance are about $2 \times 10^{-3}\mathcal{L}_{\odot}$. Given results suggest the futher work in directionWe hope that the futher development of the dynamo theory and the model elaboration will allow to make more solid conclusions about influence of the nonlinear dynamo pr changes of the global parameters of the Sun refinement

REFERENCES

- Baliunas S.L.,Jastrow R., Nature,348, p.520 (1990).
- Beer J., Tobias S., Weiss N., Sol.Phys., 181, 237(1998).
- Brandenburg, A. and Subramanian, K., 2004, Astrophysical magnetic fields and nonlinear dynamo theory, astro.ph.5052B.
- Eddy, J. A., Science, 192,1189(1976).
- Kitchatinov L.L., Ruediger G. , A& A, 276, 96(1993).
- Kitchatinov L.L., Ruediger G. , Kueker, A& A, 292, 125 (1994).
- Kitchatinov L.L., Mazur M.V., Jardine M., A&A, 359, 531(2000).
- Kueker M., Arlt R. & Ruediger G., A&A, 343, 977(1999).
- Kitchatinov L.L., Pipin V.V., Ruediger G., Astron.Nachr., v.315, p.157-163(1994).
- Kitchatinov L. L., A&A, 394,1135(2002).
- Kueker, M.; Ruediger, G.; Pipin, V. V., A&A, v.312, p.615-623(1996).
- Kuzanyan K.~M., Pipin V.~V. and Seehafer N., Sol. Phys., submitted (2005).
- Malcus W.V.R., Proctor M.R.E., J.Fluid Mech., 67,417(1975).
- Ossendrijver A.J.H., The Astron. Astrophys.Rev, 11, 287(2003).

- Ossendrijver A.J.H., Hoyng P., "Stochastic and nonlinear fluctuations in a mean field dynamo.", A&A, 313, 959(1996).
- Petrovay K., Forgaecs-dajka E., "The Role of Active Regions in the Generation of Torsional Oscillations", Sol.Phys., 205, 39(2002)
- Pipin V.V., Astronomy Rep., v.81, p.459(2004).
- Pipin V. V. and Kuzanyan K. M., ESA SPM11 proceedings (2005).
- Pipin V. , A&A, 346, 295(1999).
- Pipin, V. V., Kichatinov, L. L., Ast. Rep.,v.44, p.771(2000).
- Pipin V. V., 2005, in preparation.
- Ruediger, G. and Kitchatinov, L.L., A&A, 269, 581(1993).
- Ruediger, G. and Kitchatinov, L.L., Astron.Naschr., 321, 75 (2000).
- Rozelot J.P. "On the chaotic behaviour of the solar activity", A&A, 297, L45(1995).
- Schrijver C.J. and Harvey K.L., "The photospheric magnetic flux budget", Sol.Phys., 150, 1(1984).
- Soon, W. H., Baliunas, S. L., Zhang, Q., Sol.Phys., 154, 385(1994)
- Tobias S.M., A&A, 307, L21(1996).
- Tobias S.M., "The solar cycle: parity interactions and amplitude modulation.", A&A, 322, 1007(1997).
- Stix, M., "The Sun. An Introduction", Berlin: Springer, 2nd edition (2002).
- Weiss N.O., "Physics of the solar dynamo", In Proceedings of the International School of Physics "Enrico Fermi", G.C. Castagnoli & A. Provenzale(Eds), Amsterdam, p.325(1997).
- Yoshimura H., Kambry M.A., "The secular modulation of solar rotation from 1943 to 1992 and its time-delayed correlation with the 55-year grand cycle of the 11-year solar cycle", Sol.Phys., 148, 11(1993).